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State estimation for predictive maintenance using Kalman filter

S.K. Yang, T.S. Liu*

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu 30010, Taiwan, ROC

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Abstract

Failure can be prevented in time by preventive maintenance (PM) so as to promote reliability only if failures can be early predicted. This article presents a failure prediction method for PM by state estimation using the Kalman filter on a DC motor. An exponential attenuator is placed at the output end of the motor model to simulate aging failures by monitoring one of the state variables, i.e. rotating speed of the motor. Failure times are generated by Monte Carlo simulation and predicted by the Kalman filter. One-step-ahead and two-step-ahead predictions are conducted. Resultant prediction errors are sufficiently small in both predictions. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Kalman filter; Failure prediction; Preventive maintenance; DC motor

Nomenclature

$(\bullet)_k$:	The value of $(\bullet)$ at time kT
$(\hat{\cdot})_{a/b}$:	The estimate of $(\cdot)$ at time aT based on all known information about the process up to time bT
a:	Parallel path number for armature winding
A:	A matrix
A_c:	Coefficient matrix of the state equation for a continuous system
A_d:	Coefficient matrix of the state equation for a discrete system
A^T:	Transpose matrix of A
A^{-1}:	Inverse matrix of A
B:	Damping coefficient
B_c:	Coefficient matrix of the state equation for a continuous system
B_d:	Coefficient matrix of the state equation for a discrete system
B_k:	Coefficient matrix for the input term of a discrete state equation
C:	A matrix
C_c:	Coefficient matrix of the state equation for a continuous system
C_d:	Coefficient matrix of the state equation for a discrete system
D_c:	Coefficient matrix of the state equation for a continuous system
D_d:	Coefficient matrix of the state equation for a discrete system
E:	Applied voltage
e_b:	Motor back emf
e_f:	Field voltage
$e_k/k-1$:	Prior estimation error
E_r:	Estimation error for mean value, or confidence interval for estimated mean value
$E[X]$:	Expected value of X
$f(t)$:	Distribution function of life
H_k:	Matrix giving the ideal (noiseless) connection between the measurement and the state vector
$h(t)$:	Failure rate
i_a:	Armature winding current
i_f:	Field current
J:	Moment of inertia of rotor and load
k_1:	Motor constant
k_b:	Back emf constant
k_f:	Field flux constant
K_k:	Kalman gain
k_m:	Motor gain constant
k_T:	Motor torque constant
L_a:	Armature winding inductance
L_f:	Field inductance
L^{-1}:	The inverse Laplace transform
N:	Sample size
P:	Magnetic pole number
$P_{k/k-1}$:	Estimation error covariance matrix

* Corresponding author; fax: 886-35720634.
E-mail address: tsliu@cc.nctu.edu.tw (T.S. Liu)

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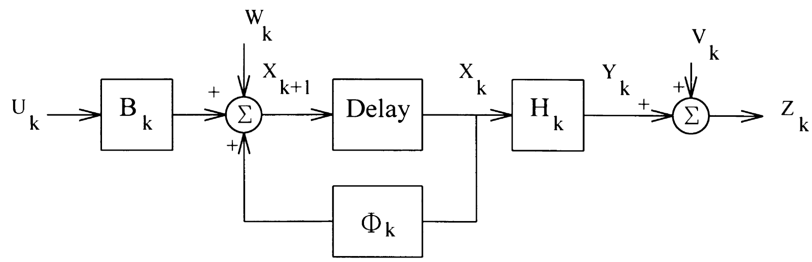


Fig. 1. Block diagram of a discrete system.

Q_k:	Covariance matrices for disturbance
R:	Armature winding resistance
R_k:	Covariance matrices for noise
t:	Time variable
T:	Motor output torque
u_i:	Standard uniformly distributed random numbers
U_k:	Control input of a discrete state equation at state k
$\mathbf{U}_k$:	Input vector at state k
V_k:	Noise, measurement error vector. It is assumed to be a white sequence with known covariance.
W_k:	Disturbance, system stochastic input vector. It is assumed to be a white sequence with known covariance and having zero crosscorrelation with V_k sequence
x, X:	Variable of a distribution function
X_{D0}:	Initial states resulting from deterministic input
X_k:	System state vector at state k
X_{S0}:	Initial states resulting from stochastic input
Y_k:	System output vector at state k
Z:	Conductor number of armature winding
Z_k:	Output measurement vector
$Z_{\alpha/2}$:	Z-value of the standard normal distribution, that resulting the cumulative probability between 0 and Z is $\alpha/2$
α:	Confidence level
θ:	Motor angle displacement
λ:	Constant failure rate
σ:	Standard deviation of a distribution function
τ:	Failure time constant of the motor
τ_m:	Motor time constant
ϕ:	Air gap flux
Φ_k:	Matrix relating X_k to X_{k+1} in the absence of a forcing function. It is the state transition matrix if X_k is sampled from a continuous process.

1. Introduction

Preventive maintenance (PM) is an effective approach to promoting reliability [1]. Time-based and condition-based maintenance are two major approaches for PM. Irrespective of the approach adopted for PM, whether a failure can be detected early or even predicted is the key point. Many methods have been proposed for failure detection in dynamic systems [2]. Fault detection based on modeling and estimation is one of the methods [3]. However, the Kalman filter is useful not only for state estimation but also for state prediction. It has been widely used in different fields during the past decades, such as on-line failure detection [4], real time prediction of vehicle motion [5], and prediction for maneuvering target trajectories [6]. The Kalman filter is a linear, discrete-time, and finite-dimensional system [7]. Its appearance is a copy of the system that is estimated. Inputs of the filter include the control signal and the difference value between measured and estimated state variables. By minimizing mean-square estimation errors, the optimal estimate can be derived. As a result, the output of the filter becomes optimal estimates of the next step time-state variables. If a device is judged to know that it is going to fail by the predicted future state variables, the failure can be prevented in time by PM. However, future state variables should be accurately predicted at a reasonably long time ahead of failure occurrence. This study proposes the state estimation and prediction for PM using the Kalman filter.

In Section 2, a discrete system model with deterministic control input and white noise disturbance and noisy output measurement will be constructed first. Equation formulation for state estimation of the Kalman filter then follows. Deterministic inputs are considered in the formulation. Moreover, equations for N-step-ahead prediction are derived. Section 3 presents the transfer function, continuous state model, and the discrete state model of a DC motor that is employed as an example in this article. Section 4 presents the simulation system with prescribed parameters, Monte Carlo simulation and ARMA model used to generate necessary data for failure prediction simulation, and the exponential attenuator used to simulate aging failure mode. Results and discussions are in Section 5.

2. Kalman filtering

2.1. System model

The block diagram of a discrete system is shown in Fig. 1. The state equations [7] are:

$$X_{k+1} = \Phi_k X_k + B_k U_k + W_k, \quad (1)$$

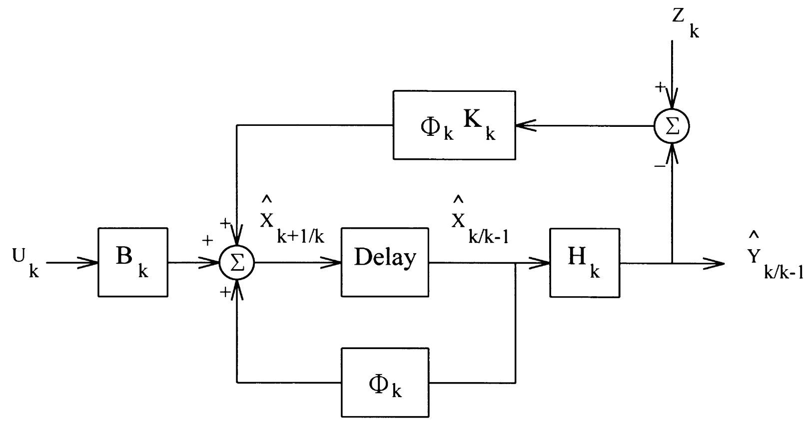


Fig. 2. Block diagram of Kalman filter.

$$Y_k = H_k X_k, \tag{2}$$

$$Z_k = Y_k + V_k. \tag{3}$$

Substituting Eq. (2) into Eq. (3) yields

$$Z_k = H_k X_k + V_k. \tag{4}$$

Let E[X] be the expected value of X, thus, covariance matrices for W_k and V_k are given by:

$$\mathrm{E}[W_k W_i^{\mathrm{T}}] = \begin{cases} Q_k, & i = k \\ 0, & i \neq k \end{cases}, \tag{5}$$

$$\mathrm{E}[V_k V_i^{\mathrm{T}}] = \begin{cases} R_k, & i = k \\ 0, & i \neq k \end{cases}, \tag{6}$$

$$\mathrm{E}[W_k V_i^{\mathrm{T}}] = 0, \qquad \text{for all } k \text{ and } i. \tag{7}$$

It follows that both Q_k and R_k are symmetric and positive definite [8].

2.2. State estimation

State estimation aims to guess the value of X_k by using measured data, i.e. $Z_0, Z_1, \ldots, Z_{k-1}$. Let $a \geq b$, and define the notation $(\hat{\cdot})_{a/b}$ as the estimate of $(\cdot)$ at time aT based on all known information about the process up to time bT. Accordingly, $\hat{X}_{k/k-1}$ is called the prior estimate of X, and $\hat{X}_{k/k}$ is called the posterior estimate of X [8]. The prior estimation error is defined as

$$e_{k/k-1} = X_k - \hat{X}_{k/k-1}. \tag{8}$$

As W_k and V_k are assumed to be white sequences, the prior estimation error has zero mean. Consequently, the associated error covariance matrix is written as

$$P_{k/k-1} = E[(e_{k/k-1})(e_{k/k-1})^{\mathrm{T}}]$$

$$= E[(X_k - \hat{X}_{k/k-1})(X_k - \hat{X}_{k/k-1})^{\mathrm{T}}]. \tag{9}$$

The estimation problem begins with no prior measurements. Thus, the stochastic portion of the initial estimate is zero if the stochastic process mean is zero; i.e. $\hat{X}_{0/-1}$ is driven by deterministic input X_{D0} only. It follows from Eq. (8) that

$$e_{0/-1} = X_0 - \hat{X}_{0/-1} = X_0 - X_{\mathrm{D0}} = X_{\mathrm{S0}}. \tag{10}$$

Employing Eqs. (9) and (10) yields

$$P_{0/-1} = \mathrm{E}[X_{\mathrm{S0}} X_{\mathrm{S0}}^{\mathrm{T}}], \tag{11}$$

where X_{D0} and X_{S0} are initial states resulting from deterministic input and stochastic input, respectively.

The Kalman filter is a copy of the original system and is driven by the estimation error and the deterministic input. The block diagram of the filter structure is shown in Fig. 2. The filter is used to improve the prior estimate to be the posterior estimate by the measurement Z_k. A linear blending of the noisy measurement and the prior estimate is written as given in Ref. [8]

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k(Z_k - H_k \hat{X}_{k/k-1}), \tag{12}$$

where K_k is a blending factor for this structure. Once the posterior estimate is determined, the posterior estimation error and the associated error covariance matrix can be derived as

$$e_{k/k} = X_k - \hat{X}_{k/k}, \tag{13}$$

$$P_{k/k} = E[(e_{k/k})(e_{k/k})^{\mathrm{T}}]$$

$$= E[(X_k - \hat{X}_{k/k})(X_k - \hat{X}_{k/k})^{\mathrm{T}}]. \tag{14}$$

The optimal blending factor is written as given in Ref. [8]

$$K_k = P_{k/k-1} H_k^{\mathrm{T}} (H_k P_{k/k-1} H_k^{\mathrm{T}} + R_k)^{-1}. \tag{15}$$

This specific K_k, namely, the one that minimizes the mean-square estimation error, is called Kalman gain.

Substituting Eq. (15) into Eq. (12), the posterior error covariance matrix can be derived as follows:

$$P_{k/k} = P_{k/k-1} - P_{k/k-1} H_k^{\mathrm{T}} (H_k P_{k/k-1} H_k^{\mathrm{T}} + R_k)^{-1} H_k P_{k/k-1}$$

$$= P_{k/k-1} - K_k (H_k P_{k/k-1} H_k^{\mathrm{T}} + R_k) K_k^{\mathrm{T}}$$

$$= (I - K_k H_k) P_{k/k-1}. \tag{16}$$

As depicted in Fig. 2, the one-step-ahead estimate is

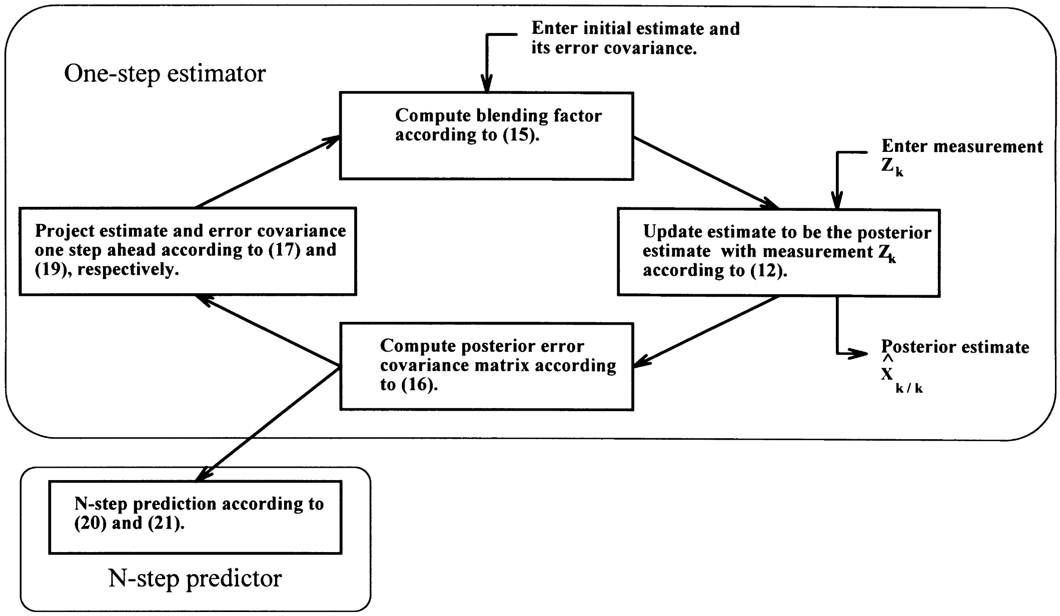


Fig. 3. One-step estimator and *N*-step predictor.

formulated as

$$\hat{X}_{k+1/k} = \Phi_k \hat{X}_{k/k-1} + \Phi_k K_k (Z_k - H_k \hat{X}_{k/k-1}) + B_k U_k$$

$$= \Phi_k [\hat{X}_{k/k-1} + K_k (Z_k - H_k \hat{X}_{k/k-1}) + B_k U_k$$

$$= \Phi_k \hat{X}_{k/k} + B_k U_k \quad (17)$$

Consequently, the one-step-ahead estimation error is derived as

$$e_{k+1/k} = (\Phi_k X_k + B_k U_k + W_k) - (\Phi_k \hat{X}_{k/k} + B_k U_k)$$

$$= \Phi_k (X_k - \hat{X}_{k/k}) + W_k$$

$$= \Phi_k e_{k/k} + W_k. \quad (18)$$

In a manner similar to Eq. (14), the one-step-ahead error covariance matrix is derived as

$$P_{k+1/k} = \mathrm{E}[(\Phi_k e_{k/k} + W_k)(\Phi_k e_{k/k} + W_k)^{\mathrm{T}}]$$

$$= \Phi_k P_{k/k} \Phi_k^{\mathrm{T}} + Q_k. \quad (19)$$

According to the aforementioned statements, several remarks for the Kalman estimation are concluded as follows:

1. As K_k is optimal, the posterior estimate $\hat{X}_{k/k}$ is an optimal estimate.
2. Based on Eqs.(12), (15), (16), (17) and (19), recursive steps for constructing an one-step estimator are summarized in Fig. 3.
3. The recursive loop has two different kinds of updating. Eqs. (12) and (16) yielding $\hat{X}_{k/k}$ and $P_{k/k}$ from $\hat{X}_{k/k-1}$ and $P_{k/k-1}$ are measurement-update; Eqs. (17) and (19) projecting $\hat{X}_{k/k}$ and $P_{k/k}$ to $\hat{X}_{k+1/k}$ and $P_{k+1/k}$ are time-update.
4. Initial conditions, i.e. $\hat{X}_{0/-1}, P_{0/-1}, \Phi_0, H_0, Q_0$, and R_0 have to be known to start recursive steps.

2.3. Prediction

The estimate resulting from recursive steps in Fig. 3 is a one-step-ahead prediction. Based on the posterior estimate, i.e. (12), the state that is *N* steps ahead of the measurement Z_k can be predicted by using the ARMA model [8]. From Eqs. (17) and (19), equations for *N*-step-ahead prediction are derived as

$$\hat{X}_{k+N/k} = \left(\prod_{i=k+N-1}^{k} \Phi_i \right) \hat{X}_{k/k} + \sum_{m=k}^{k+N-2} \left[\left(\prod_{i=k+N-1}^{m+1} \Phi_i \right) B_m U_m \right] + B_{k+N-1} U_{k+N-1}, \quad (20)$$

$$P_{k+N/k} = \left(\prod_{i=k+N-1}^{k} \Phi_i \right) P_{k/k} \left(\prod_{j=k}^{k+N-1} \Phi_j^{\mathrm{T}} \right) + \sum_{m=k}^{k+N-2} \left[\left(\prod_{i=k+N-1}^{m+1} \Phi_i \right) Q_m \left(\prod_{j=m+1}^{k+N-1} \Phi_j^{\mathrm{T}} \right) \right] + Q_{k+N-1}. \quad (21)$$

The *N*-step predictor is an appendage of the one-step estimation loop [8]. It is also shown in Fig. 3. As the current

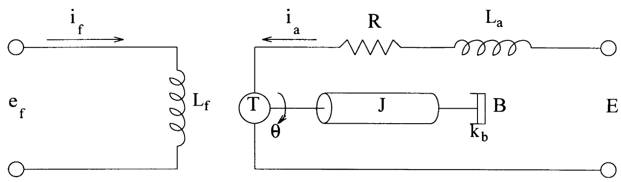


Fig. 4. Circuit representation of DC motor.

predicted value is assumed to be the initial value for the next prediction, the more steps the predictor predicts, the larger error it results in.

3. Armature-controlled DC motor

An armature-controlled DC motor is employed in this study as the physical model to perform error prediction. The motor circuit representation is shown in Fig. 4.

3.1. Transfer function

According to properties of a DC motor, the following equations can be formulated [9]:

$$\phi = k_f i_f, \tag{22}$$

$$T = \frac{ZP}{2\pi a}\phi i_a = k_1(k_f i_f)i_a = k_T i_a, \tag{23}$$

$$e_b = k_b \frac{d\theta}{dt}, \tag{24}$$

$$L_a \frac{d}{dt} i_a + R i_a + e_b = E, \tag{25}$$

$$J\ddot{\theta} + B\dot{\theta} = T, \tag{26}$$

where $k_1 = (ZP/2\pi a)$ is called the motor constant, and $k_T = k_1(k_f\ i_f)$ is the motor torque constant.

Taking the Laplace transform for Eqs. (24)–(26) results in

$$E_b(s) = k_b s\theta(s), \tag{27}$$

$$(L_a s + R)I_a(s) = E(s) - E_b(s), \tag{28}$$

$$(Js^2 + Bs)\theta(s) = T(s) = k_T\ I_a(s). \tag{29}$$

Combining Eqs. (27)–(29), the transfer function of a DC motor is derived as

$$\frac{\theta(s)}{E(s)} = \frac{k_T}{s[(sL_a + R)(sJ + B) + k_T k_b]}. \tag{30}$$

Accordingly, the block diagram of a DC motor can be shown in Fig. 5. If $L_a \approx 0$, (30) can be rewritten as

$$\frac{\theta(s)}{E(s)} = \frac{k_m}{s(s\tau_m + 1)}, \tag{31}$$

where $k_m = (k_T)/(RB + k_T k_b)$ and $\tau_m = (RJ)/(RB + k_T k_b)$ are called the motor gain constant and motor time constant, respectively.

3.2. Continuous state space model

Define $\theta, \dot{\theta}$, and i_a as state variables, so that the state vector is $X = [\theta\ \dot{\theta}\ i_a\ ]^T$. As

$$\frac{d}{dt}\theta = \dot{\theta}, \tag{32}$$

substituting Eqs. (23) and (32) into Eq. (26) yields

$$\frac{d}{dt}\dot{\theta} = \frac{1}{J}(k_T i_a - B\dot{\theta}) = \frac{k_T}{J} i_a - \frac{B}{J}\dot{\theta}. \tag{33}$$

Moreover, substituting Eq. (24) into Eq. (25) yields

$$\frac{d}{dt} i_a = \frac{1}{L_a}(E - R i_a - e_b) = \frac{E}{L_a} - \frac{k_b}{L_a}\dot{\theta} - \frac{R}{L_a} i_a. \tag{34}$$

In measurement, the rotating speed $\dot{\theta}$ is the motor output. Accordingly, continuous state equations of the DC motor are

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i_a\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & -(B/J) & (k_T/J) \\ 0 & -(k_b/L_a) & -(R/L_a)\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i_a\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ (1/L_a)\end{bmatrix}E, \tag{35}$$

$$Y = [0\ 1\ 0]\begin{bmatrix}\theta \\ \dot{\theta} \\ i_a\end{bmatrix}. \tag{36}$$

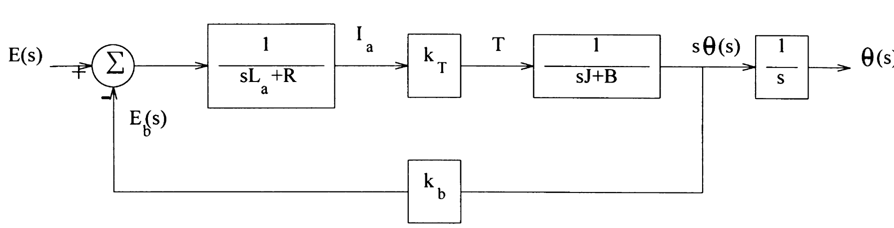


Fig. 5. Block diagram of DC motor.

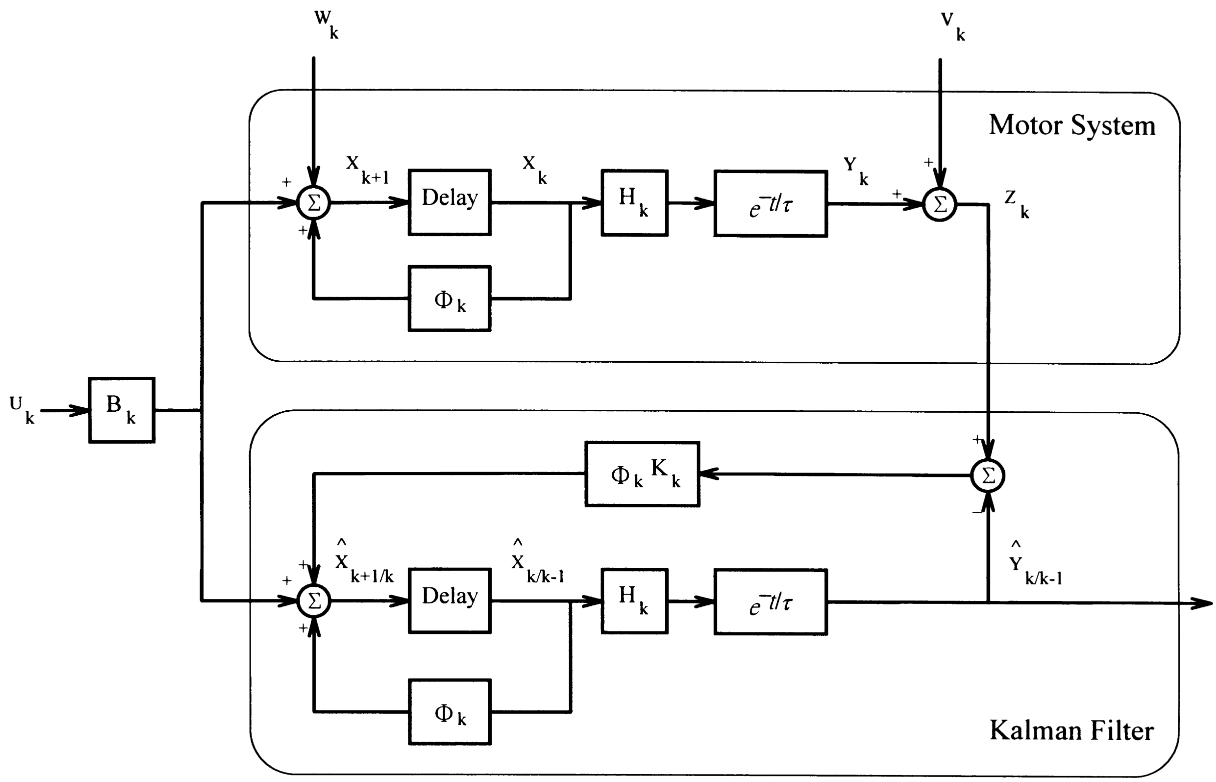


Fig. 6. Block diagram of simulation system.

3.3. Discrete state space model

The general form of state equations for a continuous system reads [10]:

$$\dot{V}(t) = A_c\, V(t) + B_c\, U(t),$$

$$Y(t) = C_c\, V(t) + D_c\, U(t). \tag{37}$$

Let $\Phi_c(t) = L^{-1}[(sI - A_c)^{-1}]$ be the state transition matrix for Eq. (37), where L^{-1} denotes the inverse Laplace transform. The discrete state equations sampled from Eq. (37) by a sample-and-hold with time interval T seconds are as follows [11]:

$$X_{k+1} = AX_k + BU_k,$$

$$Y_k = CX_k + DU_k,$$

where

$$A = \Phi_c(T), \tag{38}$$

$$B = \left[\int_0^T \Phi_c\ (\tau)\ d\tau\right] B_c, \tag{39}$$

$$C = C_c, \tag{40}$$

$$D = D_c. \tag{41}$$

4. Simulation system

4.1. Parameters

Parameters for the DC motor in this study are prescribed as follows [12]:

$$E = 10\ \text{V},\ B = 0.001\ \text{N m s},\ J = 0.01\ \text{kg m}^2,$$

$$K_T = 1\ N\ \text{m A},\ K_b = 0.02\ \text{V s},\ R = 10\ \Omega,\ L_a = 0.01\ \text{H}.$$

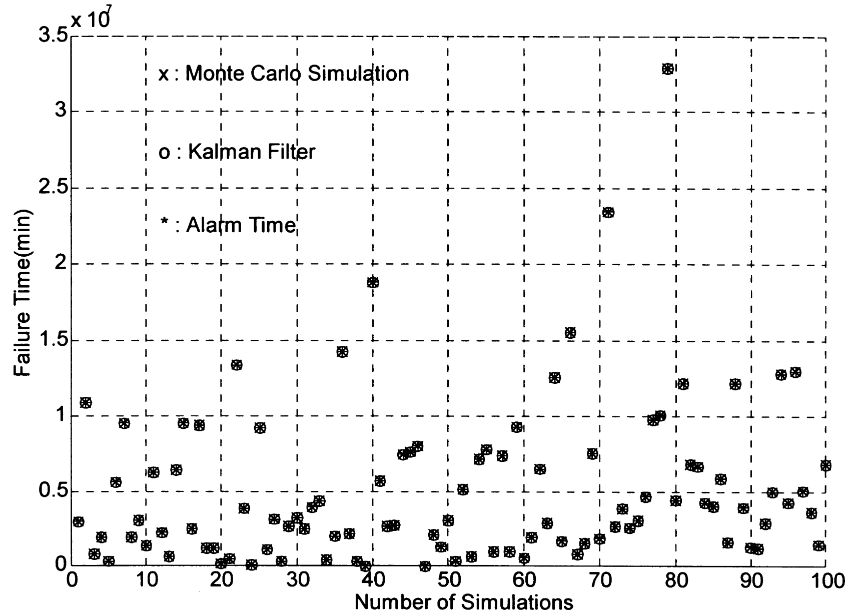
Substituting them into Eqs. (35) and (36), the continuous state equations of the motor become

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -0.1 & 100\\ 0 & -2 & -1000\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 100\end{bmatrix}10, \tag{42}$$

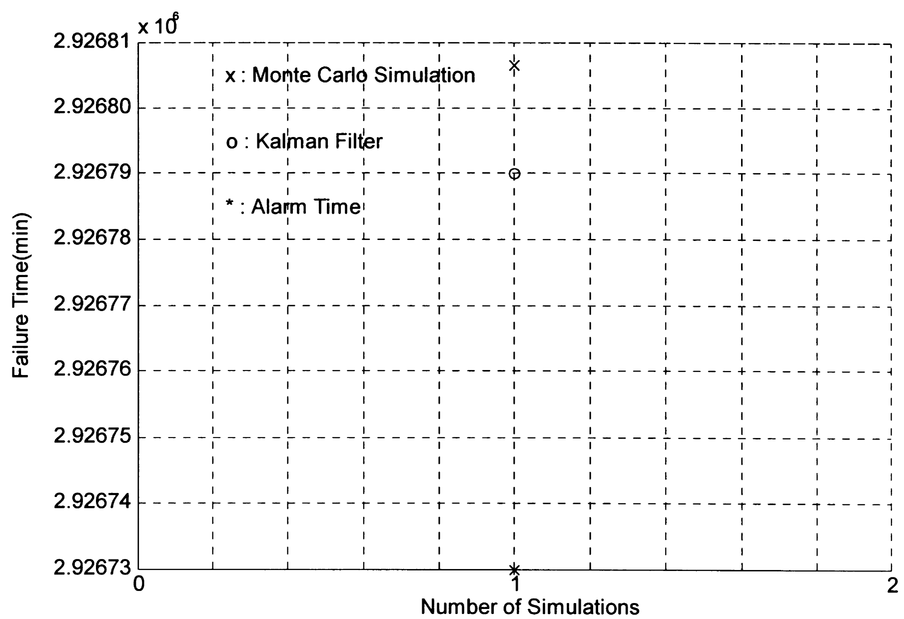
$$Y = [0\ 1\ 0]\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix}. \tag{43}$$

Besides, the following parameters are used to conduct failure prediction:

1. The failure threshold of the motor is defined as 5% less than the normal value, which is set to be the initial estimate in the Kalman prediction procedure. That is, the



(a)



(b)

Fig. 7. Failure time generated by Monte Carlo simulation and predicted by Kalman filter when lead-time = 60 min.

motor is judged to fail if the rotating speed drops to 95% of the normal value.

2. Mean time between failure (MTBF) for the motor is 100 000 h [13].
3. Sampling interval T is 1 h that is the increment time for every step in Kalman prediction.
4. Disturbance W_k has mean 0 and variance 0.01 V [14].
5. Measurement error V_k for $\dot{\theta}$ has zero mean and standard deviation of 3.333 rad s^{-1}, which is 1% full scale accuracy [15] of the measurement.
6. PM lead-time is set at $n \times 60$ min, where n is the ahead-step number for prediction. Accordingly, the alarm signal goes on for reminding PM to be executed whenever the Kalman filter predicts that the motor speed will be lower than the prescribed threshold $n \times 60$ min later.

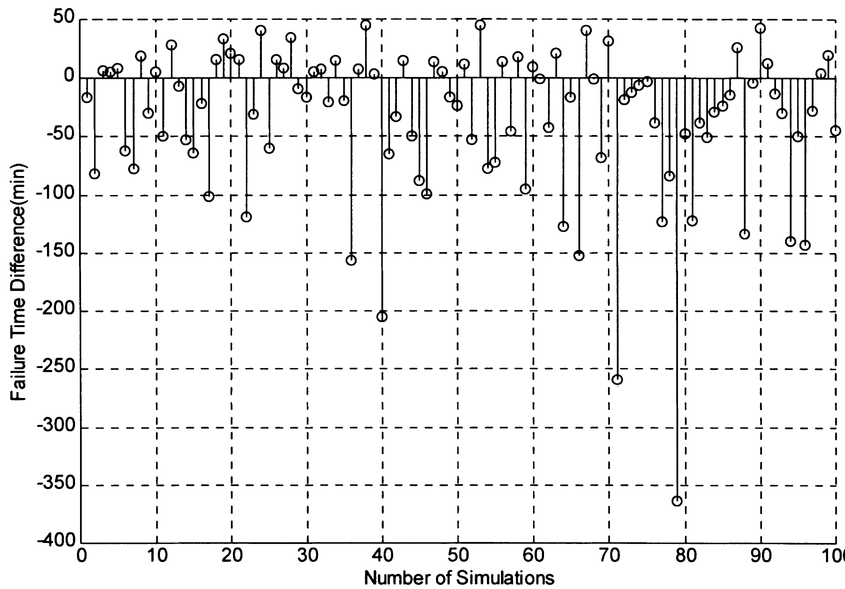


Fig. 8. Failure time difference between Monte Carlo simulation result and Kalman filter prediction when lead-time = 60 min.

4.2. Monte Carlo simulation and ARMA model

Assuming failures of the motor occur randomly. Monte Carlo simulation (MCS) is adopted to generate failure times of the motor. The relation between failure rate $h(t)$ and distribution function of life $f(t)$ is [1]

$$f(t) = h(t)\exp[-\int_0^t h(\tau)\,d\tau]. \quad (44)$$

Failures occur randomly during the useful life period of a bathtub curve [1]. The failure rate is constant during this period. Let the failure rate in (44) be a constant λ, and (44) becomes

$$f(t) = \lambda \exp[-\int_0^t \lambda\,d\tau] = \lambda\,e^{-\lambda t}, \quad (45)$$

which is an exponential distribution function. Let u_i, $i = 1,2,3,\ldots,m$, represent a set of standard uniformly distributed random numbers, the corresponding numbers t_i of the random variable t in Eq. (45), i.e. simulated failure times, are written as [1]

$$t_i = -\frac{1}{\lambda}\ln u_i, \quad (46)$$

with exponential distribution.

The measured data necessary for the recursive estimation loop of the Kalman filter, as depicted in Fig. 3, are generated by ARMA model, i.e. Eqs. (1)–(3). Simulations in this study are performed by using MATLAB [16]. All needed random numbers and white sequences with prescribed variances are obtained using the random number generator in MATLAB.

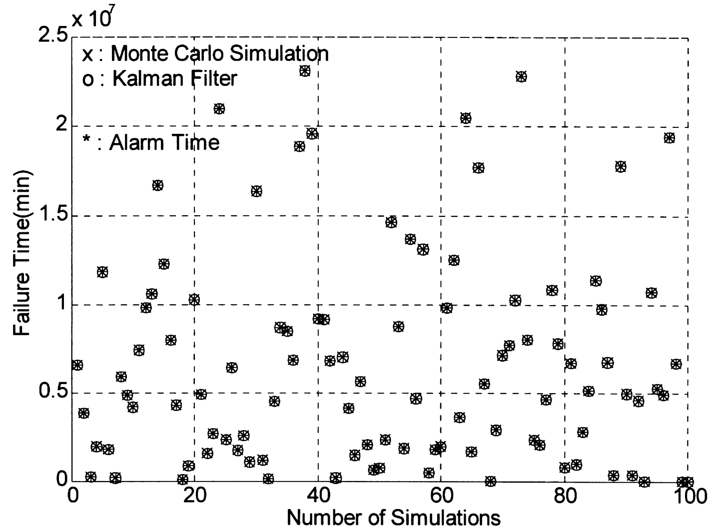
4.3. Exponential attenuator

To account for the aging failure modes and the exponentially distributed failure times t_i, an exponential attenuator, represented as $e^{-t/\tau}$, is placed at output end of both motor system and the Kalman filter. The block diagram of simulation system is shown in Fig. 6. The symbol τ of the attenuator in Fig. 6 denotes the failure time constant of the motor, which varies with failure times that are generated by MCS.

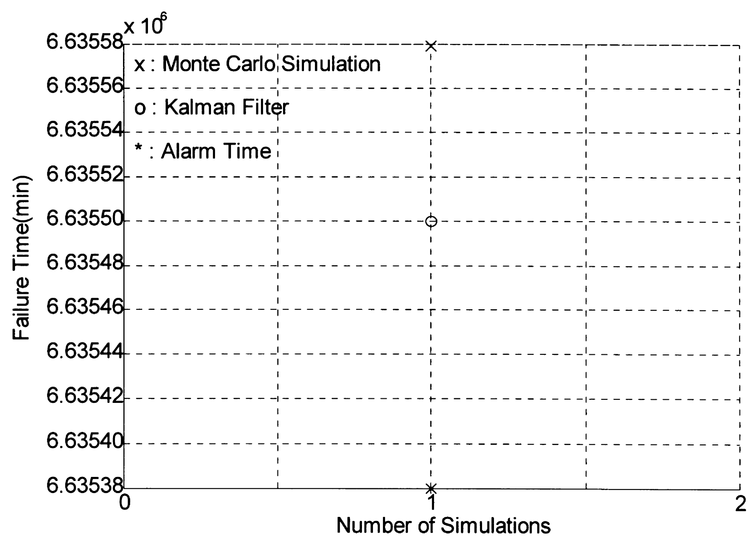
5. Results and discussions

5.1. Results

Two categories of simulation are conducted in this study, namely one-step-ahead prediction and two-step-ahead prediction. According to the central limit theorem, estimators follow the normal distribution if the sample size is sufficiently large. The sample size of 30 is a reasonable number to use [17]. The larger the sample size is, the smaller estimated error becomes, which tends to zero when the sample size approaches infinity. Hence, each simulation is executed 100 times. Simulation results for 60 min lead-time, i.e. one-step-ahead prediction, is shown in Fig. 7. Fig. 7(a) shows the results of 100 simulations of failure times generated by MCS, failure times predicted by Kalman filter, and the associated alarm times. Fig. 7(b) shows the results of one of the 100 simulations with properly scaled coordinates. The failure time differences between MCS and Kalman prediction are shown in Fig. 8. The mean value and the standard deviation of the differences for the 100 simulations are −34.71 and 65.90 min, respectively. The negative sign of the mean value indicates that the failure time predicted by Kalman filter is prior to the time generated by MCS. According to the Z formula [17], the error for estimating the mean value of the sample population can be calculated



(a)



(b)

Fig. 9. Failure time generated by Monte Carlo simulation and predicted by Kalman filter when lead-time = 120 min.

by

$$E_r^2 = \frac{Z_{\alpha/2}^2 \sigma^2}{n}.$$

The Z value for a 99% confidence level is 2.575 [17]. Solving for E_r gives

$$E_r = \frac{(2.575)(65.8954)}{\sqrt{100}} = 16.97(\text{min}).$$

According to the aforementioned data, there is 99% confidence to say that the interval for the mean value of the time difference between MCS and Kalman prediction is -34.71 ± 16.97 min, i.e. from -17.74 to -51.68 min. Taking the time difference into account, the alarm signal will appear at least 77.74 min prior to failure occurrence.

Results for the second category simulation, i.e. two-step-ahead prediction and lead-time for PM is 120 min, are shown in Figs. 9 and 10. The mean value of the failure time differences between MCS and Kalman prediction is -56.34 min, and the 99% confidence interval for this mean is 20.06 min. The maximum prediction error for this case is 76.40 min, which is 1.48 times greater than the error of the one-step-ahead prediction.

5.2. Discussions

1. In order to avoid false alarm, the failure threshold cannot be set too close to the normal value. Otherwise, a

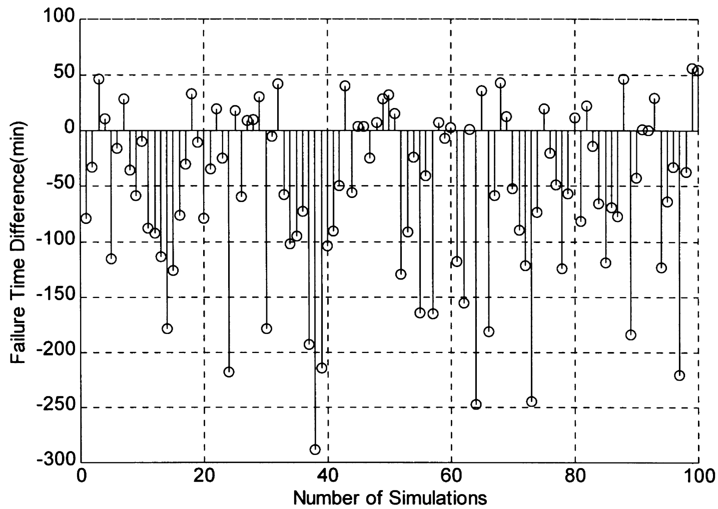


Fig. 10. Failure time difference between Monte Carlo simulation result and Kalman filter prediction when lead-time = 120 min.

decision-making algorithm is needed to identify that a failure indeed occurs.

2. The disturbance amplitude should be composed of all possible uncertainties of the motor and the environment.
3. The proposed method can not deal with abrupt changes during a sampling interval. Thus, the sampling interval should not be too long.
4. As the prediction is for PM purpose, the prediction time should be reasonably long enough for the PM action.
5. In contrast to the deterministic portion, the variance that is driven by the disturbance of the system is small. The difference of state variables between prediction steps fades very fast. Thus, using the *N*-step predictor, i.e. (20), only prediction result of the first several steps is of significance.
6. The proposed method in this study is exemplified by a motor system, which is treated as a component. The procedure can be executed on a multi-component system if state equations for the components as a whole can be constructed. Performing the procedure on either the multi-component system or each of the components are both feasible. For a complicated or large system, the proposed method can be performed on those elements in minimum cut sets that are constructed by fault tree analysis or Petri net model for failure [18].
7. Regarding multiple failure modes, they can be modeled to become modules, such as an attenuator for simulating aging failure mode for an electrical motor exemplified in this article, and placed at the system model output end to extend the proposed method. As depicted previously, the system model may be single-component or multi-component. Whether the failure modules are placed in serial parallel or other forms can be determined by system failure analysis [18]. As for a multi-component system with multiple failure modes the system can be taken apart to several components and placed the related failure module at the output end of each component to perform state estimation by Kalman filter for each component.

6. Conclusions

Failure prediction simulation for PM by state estimation through Kalman filtering has been performed in this paper. Resultant prediction errors are acceptable not only for one-step-ahead prediction but also for two-step-ahead prediction. Considerations for determining the required PM lead-time and the sampling time contradict to each other. How to compromise them and end up with an optimal value is important. To simulate the aging failure mode, a state variable, i.e. rotating speed, is monitored in this study. The more variables are measured, the more complicated failure modes can be simulated. Incorporating with fault tree analysis or Petri net model for failure, the proposed method can be performed on those elements in minimum cut sets of a complicated or large system instead of on all elements of the whole system. Failure can be prevented in time so as to promote reliability only if failures can be early predicted.

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